Logarithmic Functions and Simple Interest

Finite Math

8 January 2019

Using Properties of Exponents and Logarithms

Example

Solve for x in the following equations:

(a) $7 = 2e^{0.2x}$

(b) $16 = 5^{3x}$

(c) $8000 = (x - 4)^3$

Reminder of Some Exponent Types

A quick reminder of different types of exponents:

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Now You Try It!

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Solve for x in the following equations:

(a) $75 = 25e^{-x}$

(b) $42 = 7^{2x+3}$

(c) $200 = (2x - 1)^5$

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Solution

(a) $x \approx -1.09861$

(b) $x \approx -0.53961$

(c) $x \approx 1.94270$

Applications

Recall that exponential growth/decay models are of the form

$$A = ce^{rt}.$$

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Example

The isotope carbon-14 has a half-life (the time it takes for the isotope to decay to half of its original mass) of 5730 years.

(a) *At what rate does carbon-14 decay?*

(b) *How long would it take for 90% of a chunk of carbon-14 to decay?*

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Suppose you deposit $\$2,000$ *into a savings account with an annual simple interest rate of* 6%. *How much interest will accrue after* 6 *months?*

Future Value

Often, we might be more curious about how much will be in the account or how much will be owed on the loan after a certain period. This amount is called the *future value*. Another name for principal is *present value*. It is found by simply adding the original investment/loan amount to the interest accrued.

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Suppose you take out a $\$10,000$ *loan at a simple annual interest rate of* 3.2%. *How much would be due on the loan after* 10 *months?*

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$\$3,011.25$

Solving for Other Details

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Example

You're looking to invest $\$5,000$ *and make* $\$100$ *in interest after* 10 *weeks. What annual rate on your investment will you need to accomplish this?*

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Example

You invest $\$4,000$ *at an annual rate of* 3.9%. *How long will it take for the investment to be worth* $\$5,000$? *Give your answer in years, correct to* 2 *decimal places.*

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Solution

6.41 *years*

Commission Schedules

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Suppose a brokerage firm uses the following commission schedule

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An investor purchases 450 *shares of a stock at* \$21.40 *per share, keeps the stock for* 26 *weeks, then sells the stock for* \$24.60 *per share. What was the annual interest rate earned on the investment?*

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Example

Suppose a brokerage firm uses the following commission schedule

Principal	Commission
Under $3,000	$32+1.8% of principal
$3,000 - $10,000	$56+1% of principal
Over $10,000	$106+0.5% of principal

An investor purchases 75 *shares of a stock at* $37.90 *per share, keeps the stock for* 150 *days, then sells the stock for* $41.20 *per share. What was the annual interest rate earned on the investment? (Again, assume a* 360*-day year.)*

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Solution

6.352%

Average Daily Balance

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Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30*-day billing cycle is* \$523.18 *and purchases of* \$147.98 *and* \$36.27 *are posted on days* 12 *and* 25*, respectively, and a payment of* \$200 *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

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Example

A credit card has an annual interest rate of 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

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Solution

\$708.92

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Suppose $\$5,000$ *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

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- i = *rate per compounding period*
- n = *total number of compounding periods*

Compound Interest

Alternately, one can reinterpret this formula as a function of time as

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

where A, P, r, and m have the same meanings as above and t is the time in years.

Compound Interest

Example

If $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*